JOURNAL OF AIRCRAFT
Vol. 47, No. 1, January–February 2010

Effects of Rolling Maneuver on Divergence and Flutter of Aircraft Wing Store

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DOI: 10.2514/1.40463

The effects of the rolling maneuver on the static and dynamic aeroelastic instabilities of a cantilever wing with an external store are investigated. The structural model is based on the Goland wing and unsteady aerodynamic pressure loadings are considered. To accurately consider the spanwise location and properties of the attached external mass, the generalized function theory is used. The aeroelastic governing equations are proposed using Hamilton's variational principle and include coupling between the roll angular velocity of the maneuver and the wing elastic degrees of freedom. The solution to the resulting partial differential equations is followed by an application of the extended Galerkin method. Numerical simulations are validated against the exact flutter speed of the Goland wing and available theoretical and experimental results. In addition, the effects of the roll angular velocity, sweep angle, and store mass and location on the wing divergence and flutter are illustrated.

Nomenclature

A	=	wing cross section
ab	=	distance between the reference axes and the elastic axes
a_0	=	lift curve-slope coefficient
c	=	wing chord
$E_p^{(s)}$	=	distance between the store center of gravity and the elastic axes
e	=	distance between the aerodynamic center and the elastic axes
f_f	=	flutter frequency
g	=	gravity acceleration
h	=	plunging displacement
I_α	=	mass moment of inertia per unit length
k	=	reduced frequency
k_p	=	store radius of gyration
l	=	wing length
$M^{(s)}$	=	store mass
m	=	mass per unit length
P	=	roll angular velocity
q_n	=	air dynamic pressure
$\dot{\mathbf{R}}$	=	speed vector of an arbitrary point
$\mathbf{R}_s$	=	store displacement vector
$\mathbf{R}_w$	=	wing displacement vector
U_i	=	displacement components
U_∞	=	airstream velocity
V_f	=	flutter speed
V_n	=	air velocity normal to the wing
X_α	=	distance between the elastic axis and the wing center of gravity
X_i	=	coordinates that are fixed on the base body of the airplane
x_i	=	wing coordinate system
δ	=	variational operator
δ_D	=	Dirac delta operator
θ	=	twist angle
θ_0	=	angle of attack of the rigid wing
Λ	=	sweep angle
ρ_∞	=	air density
υ	=	Poisson's ratio
ω	=	angular frequency of vibration
$()^{\cdot}$	=	$\partial()/\partial t$
$()_{,2}$	=	$\partial()/\partial x_2$

Received 17 August 2008; revision received 2 November 2009; accepted for publication 2 November 2009.

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I. Introduction

THE evaluation of divergence and flutter clearance for an aircraft wing hosting external stores is a major aeronautical engineering task. Weight is a primary concern in flight vehicles, as weight reduction leads to the increase in structural flexibility of the aircraft and its components. Clearly estimating the aeroelastic instabilities is critical to establishing the flight envelope of newly design aircraft. One of the first works devoted to the aeroelasticity of aircraft wings is the paper by Goland [1] on the determination of the flutter speed of a uniform cantilever wing. Goland verified the flutter speed by integrating the differential equations representing the wing motion. This work was extended by Goland and Luke considering a uniform wing with tip mass [2]. Although modeling and simulations of wing aeroelasticity were performed through the years [3–6], the study of aeroelastic instability received prominence only three decades ago [7]. Indeed, much of the last decade's efforts concentrated on uniform straight wings without external stores and in uniform flow conditions.

More recently, Housner and Stein [8] analyzed the flutter instability of swept wings in subsonic flow regimes. The aeroelastic stability of a swept wing with tip weights for an unrestrained vehicle was considered by Lottati [9]. In his work a composite wing was studied and it was observed that flutter occurs at a lower speed as compared with a clean wing configuration. Gern and Librescu [10] made a considerable effort to show the effects of externally mounted stores on static and dynamic aeroelasticity of advanced swept cantilevered wings. A nonlinear aeroelastic analysis of a complete aircraft was investigated by Patil and Hodges [11]. They validated the flutter and divergence results for a metallic wing. Qin and Librescu [12] considered the static and dynamic aeroelastic instabilities of aircraft wings in incompressible and compressible flow regimes modeling the wing as an anisotropic composite thin-walled beam. Qin et al. [13,14] also studied the aeroelastic instabilities and response of advanced aircraft wings at sub- and supersonic flight speeds. Librescu and Song [15] investigated the free vibration and dynamic response to external time-dependent loads of aircraft wings carrying eccentrically located heavy stores. Also, in this case, the wing model used was a thin-walled anisotropic composite beam. Byreddy et al. [16] studied the dynamic instabilities of an aircraft wing carrying underwing store. They focused in this work on the transonic regime. The bending-torsional flutter characteristics of an aircraft wing containing an arbitrarily placed mass under a follower force have been studied by Fazelzadeh et al. [17]. The influence of the location and magnitude of the store mass and the follower force on the flutter speed and frequency of the wing was illustrated. Several recent studies have been dealing with nonlinear flutter analysis of wing-store configurations. In these works nonlinear structural and/or dynamical terms are investigated. For example, Beran et al. [18] studied the store-induced limit-cycle oscillations using a model with full system nonlinearities, whereas Tang and Dowell [19] compared theoretical and experimental results for the flutter analysis of a wing-store configuration.

Although these works and several others addressed the problem of the wing-store aeroelasticity, the effect of the aircraft maneuvers on the wing instability has not received much attention in the literature. As will be illustrated in the following sections, the maneuver has a significant influence on the dynamic response and instability of the wing-store configuration. When the aircraft is undergoing a maneuver, such as the rolling maneuver, divergence and flutter can be negatively affected. This means that these aeroelastic characteristics can deteriorate and the safety of the aircraft is in danger. Therefore, it is of paramount importance to include maneuvering conditions in the aeroelastic design and analysis of any aircraft. Consequently, for a reliable analysis of the aircraft wings it is necessary to develop refined simulation models and tools that incorporate the effects of such maneuvers.

The effect of aircraft maneuvers on aeroelasticity is considered in very few published papers. Among them, Olsen [20] revealed new insight into the aeroelasticity and flight mechanics of flexible aircraft by obtaining and solving the equations of motion for an accelerating, rotating aircraft, whereas Sipcic and Morino [21] derived the nonlinear governing equations for elastic panels in an aircraft, executing a pull-up maneuver of a given velocity and angular velocity. They modeled the aerodynamic loads as a quasi-steady pressure loading, and the effect of the maneuver on the flutter speed and the limit-cycle amplitude was discussed for various load conditions. Meirovitch and Tuzcu [22] derived a unified approach to control the complex maneuvering behavior of a flexible airplane. They considered two flight dynamics problems, including the steady level cruise and a steady level turn maneuver. The complete nonlinear dynamical equations for the general maneuvering flexible wings with sweep and dihedral angles were developed by Fazelzadeh and Mazidi [23]. These equations, valid for an isotropic nonuniform wing, include transverse shear and warping effects. Simulations have shown that the combination of flexible structural motion and maneuver parameters affects both the natural frequencies and the instability boundary.

To add to the aforementioned bulk of literature in this field, the aeroelastic modeling and flutter and divergence study of the wings hosting an arbitrarily placed mass subjected to a roll angular velocity is considered in this study. Furthermore, discussions about the combined effects of the maneuver-induced forces and external mass in conjunction with airflow on the divergence speed and the flutter speed and frequency are presented.

The rest of the paper is organized as follows. In Sec. II basic assumptions and kinematics are supplied. In Sec. III the theoretical modeling is provided. In Sec. IV divergence and flutter instability predictions are provided, whereas numerical results and pertinent discussions are given in Sec. V. Conclusions are summarized in Sec. VI.

II. Basic Assumptions and Kinematics

An isotropic nonuniform swept wing with an external mass, shown in Fig. 1, is considered. The structural flexibility as well as the angular velocities of the maneuver is taken into account when deriving the aeroelastic governing equations. The wing model is valid for long, straight, homogeneous beams and is derived from the Goland wing model.

Because of the wing sweep angle, two coordinate systems have been used here. As shown in Fig. 1, the orthogonal axes $X_1X_2X_3$ are fixed on the airplane base body. The coordinate system $x_1x_2x_3$ is the wing coordinate system in which the x_2 axis lies in the spanwise direction. Relations between these two coordinates can be obtained by the following transformation:

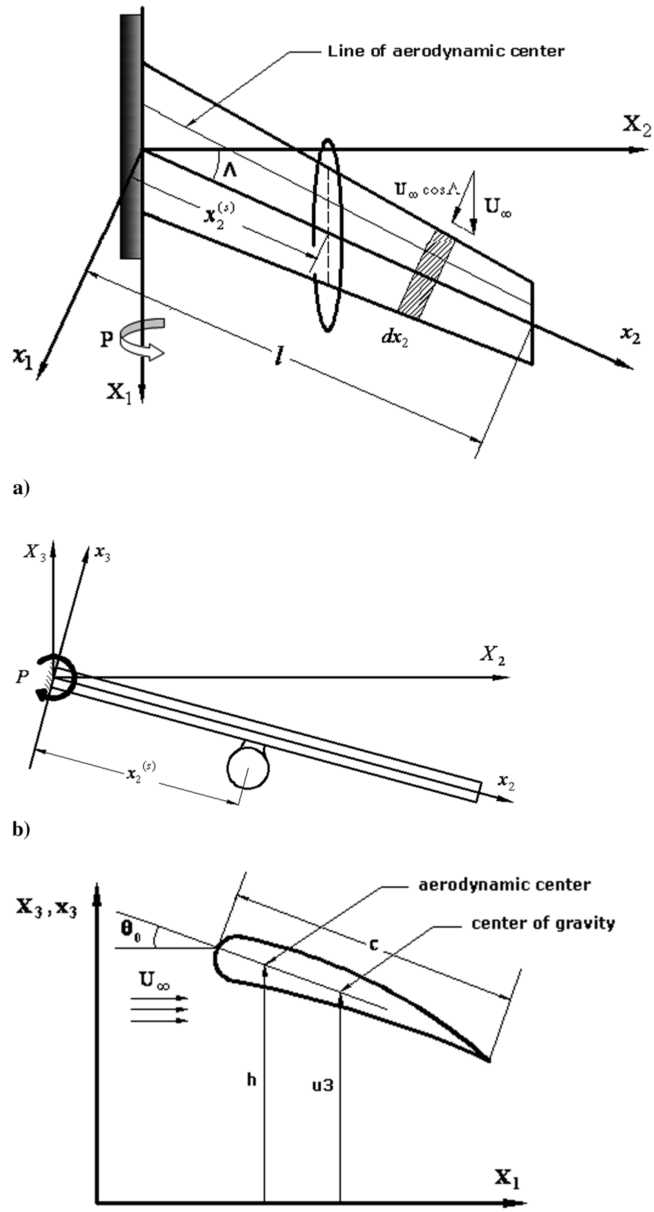


Fig. 1 Shown are the following: a) coordinate systems and geometry of a swept cantilever wing/store, b) schematic of the wing in rolling maneuver, and c) the wing typical section.

$$\mathbf{I} = \cos\Lambda\mathbf{i} + \sin\Lambda\mathbf{j} \qquad \mathbf{J} = -\sin\Lambda\mathbf{i} + \cos\Lambda\mathbf{j} \tag{1}$$

Because of the aircraft maneuver, velocity of any point on the wing or store can be obtained through transport theorem as [24]:

$$\dot{\mathbf{R}} = \frac{\partial \mathbf{R}}{\partial t} + \mathbf{P} \times \mathbf{R} \tag{2}$$

where **P** is the roll angular velocity as shown in Fig. 1. To express all equations in the wing coordinate system, the roll angular velocity must be transformed in $x_1x_2x_3$ coordinate system by means of Eq. (1):

$$\mathbf{P} = -P\mathbf{I} = -P(\cos\Lambda\mathbf{i} + \sin\Lambda\mathbf{j}) \tag{3}$$

The displacement vector of an arbitrary point on deformed wing is represented as

$$\mathbf{R}_w = [h(x_2;t) - (x_1 - ab)\theta(x_2;t)]\mathbf{k} \tag{4}$$

The position vector of the external mass is as follows:

$$\mathbf{R}_s = [h - E_P^{(s)}\theta\cos\Lambda]\mathbf{k} \tag{5}$$

where $E_P^{(s)} = x_{1_s} - ab$ is the distance between the center of gravity of the store and the wing elastic axes.

III. Aeroelastic Governing Equations

The aeroelastic governing equations are derived using Hamilton's variational principle expressed as

$$\int_{t_1}^{t_2} [\delta U_w - \delta T_w - \delta T_s - \delta W_w - \delta W_s]\,\mathrm{d}t = 0 \tag{6}$$

where U and T are the strain energy and kinetic energy and W is the work done by nonconservative forces. The indices w and s identify the wing and externally mounted store, respectively.

Applying strain-displacement relations and the generalized Hook's law, the wing strain energy is obtained. Then the first variation of the wing strain energy can be obtained as

$$\delta U_w = \int_0^l \left\{ \left[EI\frac{\partial^4 h}{\partial x_2^4}\right]\delta h - \left[GJ\frac{\partial^2 \theta}{\partial x_2^2}\right]\delta\theta \right\} \mathrm{d}x_2 \tag{7}$$

where EI and GJ are the bending and torsion stiffness, respectively. In this formulation the store is assumed to be a rigid body, implying that no additional terms have to be included into potential energy. The kinetic energy is considered next. The first variation of the wing kinetic energy is

$$\delta T_w = \int_0^l \iint_A \rho\dot{\mathbf{R}}_w.\delta\dot{\mathbf{R}}_w\,\mathrm{d}x\,\mathrm{d}A \tag{8}$$

By substituting Eqs. (2–4) into Eq. (8), this can be recast as follows:

$$\delta T_w = -\int_0^l \{[m\ddot{h} - mX_\alpha\ddot{\theta} - mP^2(h - X_\alpha\theta)]\delta h + [-mX_\alpha\ddot{h} + I_\alpha\ddot{\theta} + P^2(mX_\alpha h - I_\alpha\theta)]\delta\theta\}\,\mathrm{d}x \tag{9}$$

where m is the mass per unit length, X_α is the distance between the wing center of gravity and the elastic axes, and I_α is the mass moment of inertia per unit length about the wing elastic axes. Also, the overdot and double overdot represent the first and second derivatives in time. Furthermore, the kinetic energy of the external mass can be derived as

$$T_s = \frac{1}{2}\iint_{A_s}\int_0^l m_S(\dot{\mathbf{R}}_s.\dot{\mathbf{R}}_s)\delta_D(x_2 - x_2^{(s)})\,\mathrm{d}x_2\,\mathrm{d}A \tag{10}$$

where A_s is the store cross-sectional area. Substituting Eq. (5) in Eq. (10) and taking the first variation results in

$$\delta T_s = -M^{(s)}\int_0^l \delta_D(x_2 - x_2^{(s)})\left\{\iint_{\tau_s} [(\ddot{h} - E_P^{(s)}\ddot{\theta}\cos\Lambda - P^2(h - E_P^{(s)}\theta\cos\Lambda))\delta h + (K_P^2\ddot{\theta}\cos^2\Lambda - E_P^{(s)}\ddot{h}\cos\Lambda + P^2(E_P^{(s)}h\cos\Lambda - K_P^2\theta\cos^2\Lambda))\delta\theta]\,\mathrm{d}\tau\right\}\mathrm{d}x_2 \tag{11}$$

where K_P denotes the store radius of gyration and $M^{(s)}$ is the store mass. The virtual work of nonconservative forces acting on the wing may be expressed as

$$\delta W_w = \int_0^l (-L\delta h + M\delta\theta)\,\mathrm{d}x_2 \tag{12}$$

where L and M are the aerodynamic lift and moment, respectively. These aerodynamic loads will be discussed in next section. The first variation of the store body forces is

$$\delta W_S = M^{(s)}g\int_0^l [\delta_D(x_2 - x_2^{(s)})(-E_P^{(s)}\cos\Lambda\delta\theta + \delta h)]\,\mathrm{d}x_2 \tag{13}$$

Substituting Eqs. (7), (9), and (11–13) in Eq. (6) and noticing that for every admissible variation (δh, $\delta\theta$) the coefficient of these variations must be zero, the aeroelastic governing equations are obtained:

$$EI\frac{\partial^4 h}{\partial x_2^4} + m\frac{\partial^2 h}{\partial t^2} + mX_\alpha\frac{\partial^2\theta}{\partial t^2} + \delta_D(x_2 - x_2^{(s)})M^{(s)}\left[\frac{\partial^2 h}{\partial t^2} - E_P^{(s)}\frac{\partial^2\theta}{\partial t^2}\cos\Lambda - P^2(h - E_P^{(s)}\theta\cos\Lambda) + g\right] = -L \tag{14}$$

$$-GJ\frac{\partial^2\theta}{\partial x_2^2} + mX_\alpha\frac{\partial^2 h}{\partial t^2} + I_\alpha\frac{\partial^2\theta}{\partial t^2} + \delta_D(x_2 - x_2^{(s)})M^{(s)}\left[-E_P^{(s)}\frac{\partial^2 h}{\partial t^2}\cos\Lambda + K_P^2\frac{\partial^2\theta}{\partial t^2}\cos^2\Lambda + P^2E_P^{(s)}h\cos\Lambda - K_P^2P^2\theta\cos^2\Lambda - gE_P^{(s)}\cos\Lambda\right] = M \tag{15}$$

In these equations the Dirac delta function is used to identify the location and properties of the attached mass.

Moreover, assuming that the wing can be represented by a cantilever beam, the boundary conditions are as follows. At $x_2 = 0$, that is, at the root of wing, the deflection and slope are both zero (clamped end). At $x_2 = L$, that is, at the wing tip, the moment and shear forces are both zero (free end). By using these boundary conditions, the aeroelastic governing equations will be solved.

IV. Divergence and Flutter Solution Methodology

Because of the intricacy of the aeroelastic governing equations, the solution is sought by using an approximate solution procedure. To this end, h and θ are represented as

$$h = \sum_{i=1}^{N}\bar{h}_i\eta_i e^{i\omega t}, \qquad \theta = \sum_{i=1}^{N}\bar{\theta}_i\eta_i e^{i\omega t} \tag{16}$$

where η denotes the nondimensional spanwise coordinate represented as $\eta = x_2/l$ and N is number of modes. Six bending modes and six torsion modes are considered to transform the governing equations into a set of first-order coupled ordinary differential equations. The following family of orthogonal functions for $\bar{h}$ and $\bar{\theta}$ is used here [25]:

$$\bar{h}_i = \frac{(x/l)^{1+i}\{6 + i^2(1 - x/l)^2 + i[5 - 6x/l + (x/l)^2]\}}{i(1+i)(2+i)(3+i)}$$

$$\bar{\theta}_i = \sin\left(\frac{i\pi x}{l}\right) \tag{17}$$

The governing Eqs. (14) and (15) include aerodynamic lift and moment that are derived from Theodorsen's unsteady thin-airfoil theory. These are

$$L(x_2,t) = -\pi\rho_\infty\omega^2 b^3\left[\frac{h}{b}L_{hh} + \frac{\partial h}{\partial x_2}L_{hh'} + \theta_{\text{eff}}L_{h\theta} + b\frac{\partial\theta_{\text{eff}}}{\partial x_2}L_{h\theta'}\right] \tag{18}$$

$$M(x_2,t) = -\pi\rho_\infty\omega^2 b^4\left[\frac{h}{b}M_{\theta h} + \frac{\partial h}{\partial x_2}M_{\theta h'} + \theta_{\text{eff}}M_{\theta\theta} + b\frac{\partial\theta_{\text{eff}}}{\partial x_2}M_{\theta\theta'}\right] \tag{19}$$

where b is the wing semi-chord and $L_{hh}, L_{hh'}, \ldots, M_{\theta\theta'}$ are the aerodynamic coefficients that can be found in Hodges and Pierce [25]. The Theodorsen function $C(k)$ appearing in these coefficients and used in the following analysis is

$$C(k) = F(k) + iG(k) = \frac{0.021573 + 0.210400k + 0.512607k^2 + 0.500502k^3}{0.021508 + 0.251239k + 1.035378k^2 + k^3} - i\,\frac{0.001995 + 0.327214k60.122397k^2 + 0.000146k^3}{0.089318 + 0.934530k + 2.48148k^2 + k^3} \tag{20}$$

where k is the reduced frequency expressed as $k = \omega b/V_n = \omega b/(U_\infty \cos\Lambda)$. Also, θ_{eff} is the effective sectional angle of attack given by

$$\theta_{\text{eff}} = \theta_0 + \theta - (Px_2/V_n) \tag{21}$$

where θ_0 denotes the angle of attack of the rigid wing. Furthermore, it is assumed that the aerodynamic loading on the store can be neglected.

Employing these equations into governing equations (14) and (15) finally leads to a complex eigenvalue problem expressed in matrix form as

$$\mathbf{A} - \omega^2\mathbf{B} = 0 \tag{22}$$

where **A** denotes the (real) stiffness matrix of the wing and **B** is the (complex) matrix representing the inertia terms of the wing and external mass as well as the complex aerodynamic parameters. The real part of the complex valued quantity ω represents the circular frequency of the oscillation, whereas its imaginary part constitutes the damping factor.

For the static case, to find the divergence speed, the determinant of the matrix **A** should fulfill the condition $|\mathbf{A}| = 0$. The corresponding smallest positive value of air speed represents the divergence speed. The implemented dynamic solution methodology, as discussed in [10], is based on the inversion of the complex matrix **B** and the subsequent calculation of complex eigenvalues and eigenvectors of the obtained system matrix $\mathbf{AB}^{-1}$. The flutter speed is calculated in a converging iteration process by rendering zero the imaginary (damping) part of the complex eigenvalues.

Table 1 Characteristics of the Goland wing model

Parameters	Value
Wing length	6.1 m
Semichord	0.915 m
Bending rigidity	9.765×10^6 N · m²
Torsional rigidity	0.989×10^6 N · m²
Mass per unit length	35.695 kg/m
Moment of inertia	8.694 kg · m
X_a	0.183 m
a	−0.34

V. Numerical Results

To demonstrate the effects of the wing sweep angle, store mass and location, and variation of the roll angular velocity on the flutter speed and frequency and divergence speed of cantilever wings, a wing with a uniform rectangular cross section from [1] is considered. The wing characteristics are presented in Table. 1. Also, the following nondimensional parameters are used:

$$\eta_S = x_2^{(S)}/l, \qquad \varepsilon_S = E_P^{(S)}/C$$
$$\xi_S = M_{\text{store}}/M_{\text{wing}}, \qquad \bar{P} = P/\omega_h$$

where $\omega_h = EI/ml^4)^{1/2}$ and EI is the bending rigidity. It should be noted that some of the results reported include large values of roll angular velocity to better identify its contribution to the aeroelastic behavior of the wing during the maneuver.

A. Theoretical and Experimental Validations

For model validation purposes the results for the wing without external mass and rolling maneuver are compared in Table 2 with several theoretical published papers. Flutter speed and flutter frequency results are presented in this table and excellent agreement is achieved. Furthermore, in Table 3 for the purpose of validation the results in the absence of the roll angular velocity are compared with theoretical and experimental results of [6] for different spanwise locations of the external mass. Also, in this case, good agreement with theoretical and experimental results is observed.

B. Numerical Solutions for the Divergence

The results related to the static aeroelastic response of the wing are considered next. Figure 2 shows a parametric study investigating the

Table 2 Comparison of calculated flutter speed and frequency for a uniform cantilevered clean wing

Method	Flutter speed, km/h	Flutter frequency, Hz
Exact (Goland [1])	494.1	11.25
Housner and Stein [8]	483.1	11.27
Gern and Librescu [10]	493.6	12.02
Patil and Hodges [11]	488.3	11.17
Qin and Librescu [12]	493.2	11.15
Present	494.0	11.13

Table 3 Validation of flutter speed and frequency for wing/store

	Theoretical [6]		Experimental [6]		Present					
Location of the store, m	V_f, m/s	f_f, Hz	V_f, m/s	f_f, Hz	V_f, m/s	Error respect to theoretical, %	Error respect to experimental, %	f_f, Hz	Error respect to theoretical, %	Error respect to experimental, %
0	101.50	25.27	101.80	22.10	98.81	−2.65	−2.9	24.56	−2.81	11.1
0.2794	100.89	19.23	98.75	17.40	96.05	−4.80	−2.7	19.56	1.72	12.4
0.4318	124.05	28.04	116.43	26.80	119.93	−3.32	3.01	28.18	0.50	5.15
0.7620	160.32	30.68	——	——	159.5	−0.51	——	30.83	0.49	——
1.1430	122.22	25.67	——	——	119.95	−1.86	——	26.09	1.64	——
1.1684	112.17	24.87	112.17	21.80	108.10	−3.63	−3.6	25.46	2.37	16.8
1.2192	91.44	23.60	97.54	21.40	93.62	2.38	−4.0	24.35	3.18	13.8

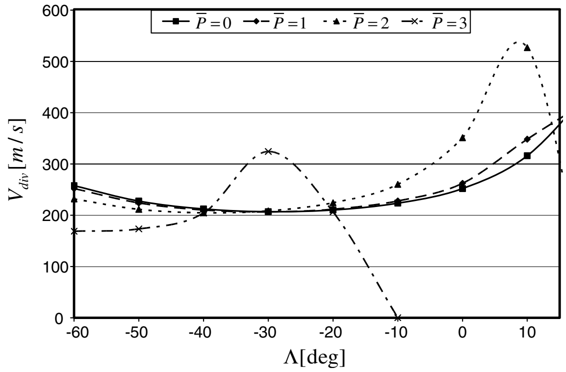
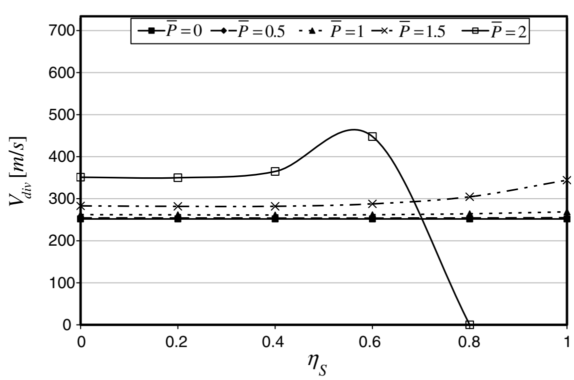


Fig. 2 Effects of sweep angle on the wing divergence speed for selected values of the nondimensional maneuver angular velocity.

effect of the sweep angle on the divergence speed of the clean wing for selected values of the maneuver angular velocity. Result trends for the nonmaneuvering case are the same as those reported in previous papers [9,10]. The total effect of sweep depends strongly on whether the wing is swept back or forward. It is apparent that forward sweep causes the wing to be more susceptible to divergence, whereas backward sweep increases the divergence speed. In Fig. 2, the forward swept wings are considered. Indeed, a small amount of backward sweep rapidly increases the divergence speed [25]. However, as it can be inferred from this figure, for large negative values of the sweep angle, the increase in the maneuver angular velocity will decrease the divergence speed. Furthermore, when the sweep angle increases the divergence speed also increases. This behavior continues to a peak value for every chosen value of the maneuver angular velocities; however, when the sweep angle increases further, it is peculiar that the divergence speed decreases dramatically until the divergence speed goes to zero. This fact is obvious for $\bar{P} = 3$ in this figure. This can be qualitatively explained as the increase of the destabilizing effect of the maneuver induces forces leading to instability, even at zero air speed.

The influence of the store mass on the divergence speed of the wing is shown in Fig. 3. The divergence speed is computed as a function of the store spanwise location for two values of the store mass and for different maneuver angular velocities. These maneuver angular velocities are lower than those considered in Fig. 2, because the destabilizing effect of the external mass causes the divergence to occur at lower values of the roll angular velocity. Interestingly, for $\bar{P} = 0$ the store spanwise location does not affect the divergence speed. This is because in the nonmaneuvering case the stiffness matrix does not contain any effects of the store. Similar to the previous case, for every choice of the maneuver angular velocity, moving the external mass toward the wing tip yields zero air speed divergence. This occurs at different store spanwise locations depending on the roll angular velocity. The figure also shows that, as the store mass becomes greater in case b, the zero air speed divergence takes place earlier than case a, when moving the external mass toward the wing tip. Therefore, the allowable spanwise region in which the store can be safely mounted is limited, that is, the store should be located on the wing between the root and a specific spanwise location away from the location that might result in zero divergence flight speed.

The influence of the wing sweep angle on the divergence speed is shown in Fig. 4 for two different spanwise locations of the external mass and for selected values of the roll angular velocity. In Fig. 4a the external mass is located at the midspan of the wing, whereas in Fig. 4b the tip mass with the same mass is considered. The figure shows that for large values of the roll angular velocity the divergence takes place at zero air velocity. As discussed earlier, this is due to the destabilizing effect of the dynamic induced forces added to the system. A comparison of Figs. 4a and 4b shows that these zero air velocity divergence points depend on the location of the mass. In a



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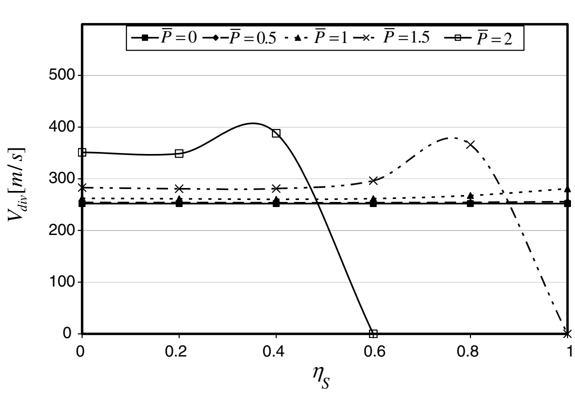
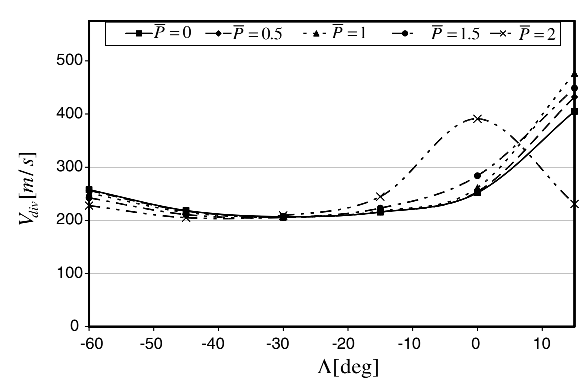


b)

Fig. 3 Effects of the spanwise location of the store on the wing divergence speed for selected values of the nondimensional maneuver angular velocity: a) $\xi_S = 0.25$, $\varepsilon_s = 0.1$; and b) $\xi_S = 0.5$, $\varepsilon_s = 0.1$.



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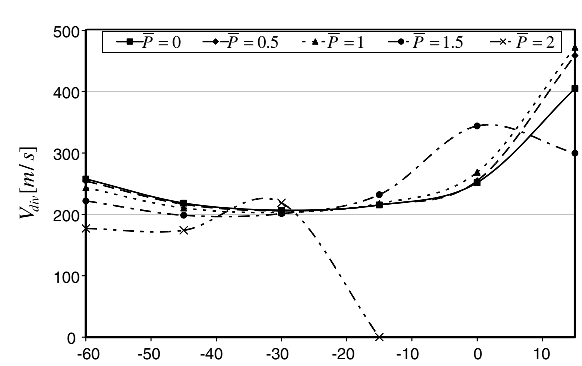
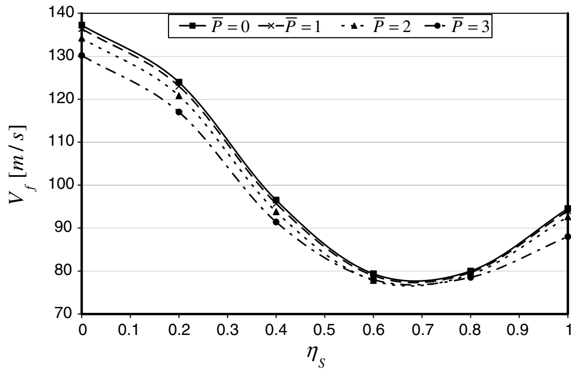
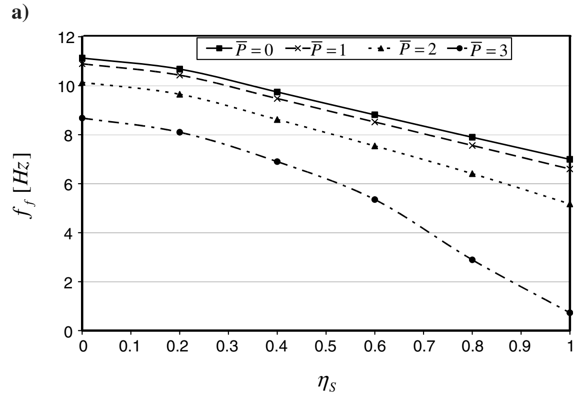


b)

Fig. 4 Effects of the sweep angle on the wing divergence speed for selected values of the nondimensional maneuver angular velocity: a) $\xi_S = 0.25$, $\varepsilon_s = 0.1$; b) $\xi_S = 0.25$, $\varepsilon_s = 0.1$, $\eta_S = 1$.

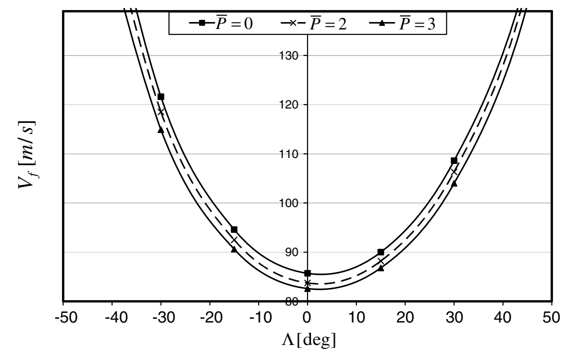


a)

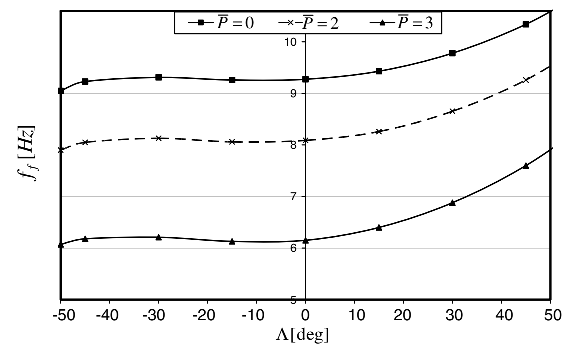


b)

Fig. 5 Effect of the spanwise location of the store for selected values of the nondimensional roll angular velocity and $\xi_S = 0.25$, $\varepsilon_s = 0.1$: a) flutter speed, and b) flutter frequency.

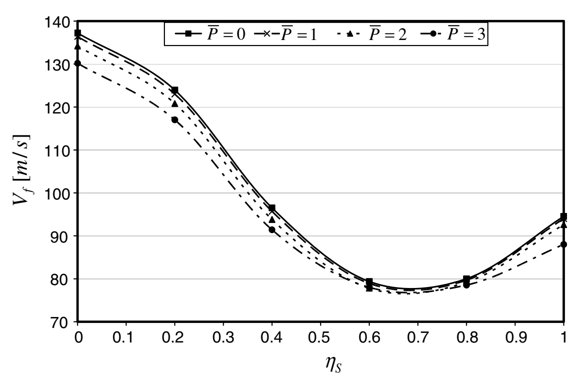


a)

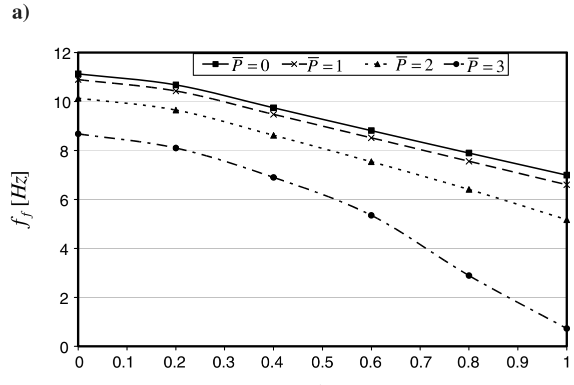


b)

Fig. 7 Effect of the wing sweep angle for selected values of the nondimensional roll angular velocity and $\xi_S = 0.25$, $\varepsilon_s = 0.1$, $\eta_S = 0.5$: a) flutter speed, and b) flutter frequency.

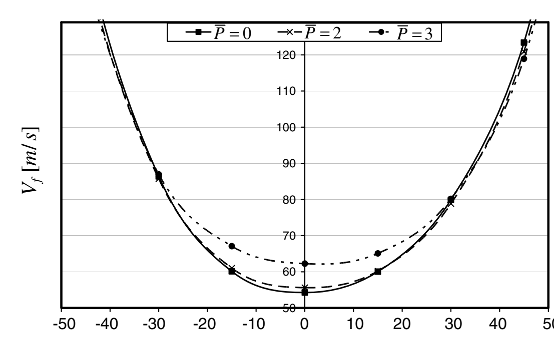


a)

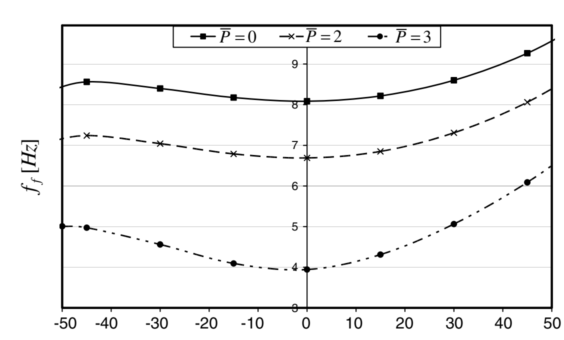


b)

Fig. 6 Effect of the spanwise location of the store for selected values of the nondimensional roll angular velocity and $\xi_S = 0.5$, $\varepsilon_s = 0.1$: a) flutter speed, and b) flutter frequency.



a)



b)

Fig. 8 Effect of the wing sweep angle for selected values of the nondimensional roll angular velocity and $\xi_S = 0.5$, $\varepsilon_s = 0.1$, $\eta_S = 0.5$: a) flutter speed, and b) flutter frequency.

static case, moving the mass toward the wing tip will reduce the aeroelasticity stability region of the wing.

C. Numerical Solutions for the Flutter Speed

This subsection focuses on flutter simulations. Figure 5 shows the variation of the flutter speed and frequency of the wing for selected values of the roll angular velocity due to variations in the spanwise location of the external mass. The effect of the roll angular velocity on the wing flutter speed is clearly highlighted. The results show that an increase of roll angular velocity can induce a lower flutter speed. This means that the rolling maneuver decreases the maneuvering ability of the airplane. Figure 6 reveals the same results for a heavier external mass. The store mass is doubled in this case. The effect of the spanwise location of the external mass on the flutter velocity and the flutter frequency for selected values of the roll angular velocity is emphasized. Clearly, the effect of the roll angular velocity on the flutter speed is strongly dependent on the store mass and the store location.

Figures 7 and 8 show the effects of the sweep angle and the roll angular velocity on the flutter speed and frequency of a cantilever wing for two different values of the external mass. Comparing the results, it appears that the store mass decreases the flutter speed and plays a destabilizing role in dynamic stability of the wing; however, both positive and negative sweep angles will increase the flutter speed. Furthermore, the effects of the roll angular velocity on the wing flutter speed and frequency can be observed. The variation of the flutter speed presented in these figures is similar to those obtained by Gern and Librescu [10] and Lottati [9]. It is also important to notice that increasing the roll angular velocity decreases the flutter frequency of the wing.

VI. Conclusions

The effect of the rolling maneuver, one of the most popular flight maneuvers, on the divergence and flutter of an airplane wing carrying an arbitrarily placed store is considered. To this end, the complete aeroelastic equations for an isotropic Goland wing with sweep angle under rolling maneuver are formulated and validated. The equations include effects of both maneuver- and flow-induced forces. These equations are valid for long, straight, homogeneous wings carrying a rigid mass.

Results show the influence of the roll angular velocity, the wing sweep angle, and the store mass on the wing aeroelastic instability boundaries. Comparing the divergence and flutter results, one can conclude that the rolling maneuver has a more profound and detrimental impact on the static divergence than on the dynamic flutter. The rolling maneuver restricts the wing dynamic stability region in most of cases. But, under certain situations, the flutter boundary can be improved by the rolling. On the other hand, increasing the rolling moment always seems to lower the flutter frequency. Also, because of the destabilizing effect of the maneuver-induced forces for large values of the roll angular velocity, the divergence takes place at zero air velocity. Furthermore, it is found that the divergence and flutter speeds in the case of a heavy store is lower than those obtained for a light one, independent of the maneuver conditions.

References

[1] Goland, M., "The Flutter of a Uniform Cantilever Wing," *Journal of Applied Mechanics*, Vol. 12, 1945, pp. 197–208.

[2] Goland, M., and Luke, Y. L., "The Flutter of a Uniform Wing with Tip Weights," *Journal of Applied Mechanics*, Vol. 15, 1948, pp. 13–20.

[3] Fung, Y. C., *The Theory of Aeroelasticity*, Dover, Toronto, 1955.

[4] Runyan, H. L., and Sewall, J. L., "Experimental Investigation of the Effects of Concentrated Weights on Flutter Characteristics of a Straight Cantilever Wing," NACA TN-1848, 1949.

[5] Eastep, F., "Transonic Flutter Analysis of a Rectangular Wing with Conventional Airfoil Sections," *AIAA Journal*, Vol. 18, 1980, pp. 1159–1164.
doi:10.2514/3.50866

[6] Harry, L. R., and Charles, E. W., "Flutter of a Uniform Wing with an Arbitrarily Placed Mass According to a Differential Equation Analysis and a Comparison with Experiment," NASA Technical Report NACA TN-1848, 1949.

[7] "Specialists Meeting on Wing-with-Stores Flutter," AGARD, AGARD-CP-162, 1975.

[8] Housner, J. M., and Stein, M., "Flutter Analysis of Swept-Wing Subsonic Aircraft with Parameter Studies of Composite Wings," NASA TN D-7539, 1974.

[9] Lottati, I., "Aeroelastic Stability Characteristics of a Composite Swept Wing with Tip Weights for an Unrestrained Vehicle," *Journal of Aircraft*, Vol. 24, 1987, pp. 793–802.
doi:10.2514/3.45523

[10] Gern, H., and Librescu, L., "Effect of Externally Mounted Stores on Aeroelasticity of Advanced Aircraft Wings," *Journal of Aerospace Science and Technology*, Vol. 2, 1998, pp. 321–333.
doi:10.1016/S1270-9638(98)80008-4

[11] Patil, M. J., and Hodges, D. H., "Nonlinear Aeroelastic Analysis of Composite Aircraft in Subsonic Flow," AIAA Paper A98-31713, 1998.

[12] Qin, Z., and Librescu, L., "Aeroelastic Instability of Aircraft Wings Modeled as Anisotropic Composite Thin-Walled Beams in Incompressible Flow," *Journal of Fluids and Structures*, Vol. 18, 2003, pp. 43–61.
doi:10.1016/S0889-9746(03)00082-3

[13] Qin, Z., Librescu, L., and Marzocca, P., "Aeroelasticity of Composite Aero Vehicle Wings in Supersonic Flows," *Journal of Spacecraft and Rockets*, Vol. 40, 2003, pp. 162–173.
doi:10.2514/2.3950

[14] Qin, Z., Marzocca, P., and Librescu, L., "Aeroelastic Instability and Response of Advanced Aircraft Wings at Subsonic Flight Speeds," *Aerospace Science and Technology*, Vol. 6, 2002, pp. 195–208.
doi:10.1016/S1270-9638(02)01158-6

[15] Librescu, L., and Song, O., "Dynamics of Composite Aircraft Wings Carrying External Stores," *AIAA Journal*, Vol. 46, 2008, pp. 568–572.
doi:10.2514/1.25541

[16] Byreddy, C., Grandhi, R. V., and Beran, P., "Dynamic Aeroelastic Instabilities of an Aircraft Wing with Under Wing Store in Transonic Regime," *Journal of Aerospace Engineering*, Vol. 18 No. 4, 2005, pp. 206–214.
doi:10.1061/(ASCE)0893-1321(2005)18:4(206)

[17] Fazelzadeh, S. A., Mazidi, A., and Kalantari, H., "Bending-Torsional Flutter of Wings with an Attached Mass Subjected to a Follower Force," *Journal of Sound and Vibration*, Vol. 323, 2009, pp. 148–162.
doi:10.1016/j.jsv.2009.01.002

[18] Beran, P. S., Strganac, T. W., Kim, K., and Nichkawde, C., "Studies of Store-Induced Limit-Cycle Oscillations Using a Model with Full System Nonlinearities," *Nonlinear Dynamics*, Vol. 37 No. 4, 2004, pp. 323–339.
doi:10.1023/B:NODY.0000045544.96418.bf

[19] Tang, D. M., and Dowell, E. H., "Flutter and Limit Cycle Oscillations for a Wing–Store Model with Freeplay," *Journal of Aircraft*, Vol. 43 No. 2, 2006, pp. 487–503.
doi:10.2514/1.12650

[20] Olsen, J. J., "Unified Flight Mechanics and Aeroelasticity for Accelerating, Maneuvering, Flexible Aircraft," *RTO AVT Specialists' Meeting on Structural Aspects of Flexible Aircraft Control*, NATO Research and Technology Organization, Neuilly-sur-Seine, France, 1999, pp. 1–12.

[21] Sipcic, S. R., and Morino, L., "Dynamic Behavior of the Fluttering Two-Dimensional Panels on an Airplane in Pull-Up Maneuver," *AIAA Journal*, Vol. 29, 1991, pp. 1304–1312.
doi:10.2514/3.10736

[22] Meirovitch, L., and Tuzcu, I., "Integrated Approach to the Dynamics and Control of Maneuvering Flexible Aircraft," NASA, CR-2003-211748, 2003.

[23] Fazelzadeh, S. A., and Mazidi, A., "Nonlinear Equations of Motion for the Maneuvering Flexible Aircraft Wings," *Proceedings of ASME 6th International Symposium FSI, Aeroelasticity, FIV and Noise*, Vol. 9, American Society of Mechanical Engineers Pressure Vessels and Piping Division, New York, July 2006, pp. 217–226.

[24] Baruh, H., *Analytical Dynamics*, McGraw–Hill, Boston, 1999.

[25] Hodges, D. H., and Pierce, G. A., *Introduction to Structural Dynamics and Aeroelasticity*, Cambridge Univ. Press, Cambridge, England, UK, 2002.